Contributions of Maxwell to Electromagnetism

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Maxwell, one of the greatest physicists of the nineteenth century, was the founder of a consistent theory of electromagnetism. However, it must be noted that significant discoveries and intelligent efforts of Coulomb, Volta, Ampére, Oersted, Faraday, Gauss, Poisson, Helmholtz and others preceded the work of Maxwell, enhancing a partial understanding of the connection between electricity and magnetism. Maxwell, by sheer logic and physical understanding of the earlier discoveries completed the unification of electricity and magnetism. The aim of this article is to describe Maxwell's contribution to electricity and magnetism.

Introduction

Historically, the phenomenon of magnetism was known at least around the 11th century. Electrical charges were discovered in the mid 17th century. Since then, for a long time, these phenomena were studied separately since no connection between them could be seen except by analogies. It was Oersted's discovery, in July 1820, that a voltaic current-carrying wire produces a magnetic field, which connected electricity and magnetism. Another significant step towards establishing this connection was taken by Faraday in 1831 by discovering the law of induction. Besides, Faraday was perhaps the first proponent of, what we call in modern language, field. His ideas of the 'lines of force' and the 'tube of force' are akin to the modern idea of a field and flux, respectively. Maxwell was backed by the efforts of these and other pioneers. Maxwell finally completed the unification of electricity and magnetism in a classical sense showing the highest form of creativity.

Keywords
Electricity history, magnetism, history, Maxwell's unification, displacement current.

Electricity and Magnetism before Maxwell

Magnetism was discovered before electricity. The Chinese knew the directive property of magnets as early as the 11th century. These magnets were natural ones and were called loadstone (Fe_3O_4). Pierre de Maricourt (1269) discovered that a piece of loadstone in the globular form had a peculiar property. He brought a small magnetic needle and marked the line along which the needle sets itself. He marked many such lines covering the entire surface of the globular loadstone. The lines drawn by Maricourt showed that these lines girdle the globular magnet, concentrating on two diametrically opposite ends, precisely similar to the way longitudes girdle the Earth. He called the two ends the two poles. Gilbert (1600) discovered that the Earth is also a giant magnet where the geographical North Pole is the magnetic south pole. Gilbert did many experiments in static electricity and proved that the attractive power of amber could be reproduced by friction in many other materials such as glass, sealing wax and sulphur. Thus the modern history of magnetism and electricity begins with Gilbert.

This attractive force was named by Gilbert as 'electric', and he conjectured that it is released from the materials by friction. The repulsion phenomenon in 'electric' was also observed and two types of 'electric' (i.e. two types of charges) were inferred. They were called vitreous (+) and resinous (–) electricity. In 1729, Gray discovered that the electric charges could be transferred from one body to the other, and that there are materials called 'non-electrics' (conductors) through which the charges could be transported. Various experiments on static electricity were performed by Musschenbroek (1745) and also around the same time by William Watson. It is very difficult to give the credit for the discovery of the principle of conservation of charge to anyone in particular but it appears that, Watson and Franklin were aware of it through their experiments. Mathematically stated, it is nothing but the consequence of the equation of continuity of electrical fluid. The analogy with fluid flow played an important role in the later developments. The same principle of conservation of charge played a pivotal role in Maxwell's theory. Joseph Priestley (1767), the discoverer of oxygen, found that like charges repel and unlike charges attract and guessed that the law of force might be inverse square. However, correct and experimentally detailed investigations of the law of force were carried out by Coulomb in 1785. Coulomb also strongly supported the 'two fluid theory', which means, in modern language, that there are two types of electricity. There was, however, no concrete mathematical theory of electrostatics. Since the law of force of attraction or repulsion was discovered to be inverse square, it was natural to compare it with gravitation. This comparison, however, is partial, because, there are no two types of masses. Lagrange in 1777 had discovered the existence of gravitational potential. Laplace had shown that in a mass-free region, the gravitational potential satisfied, the now famous Laplace's equation

$$\nabla^2 V=0.$$

Poisson (1813), in analogy with the inverse square law of Coulomb, guessed, and that too correctly, that for an electrical system, the equation for electrostatic potential V should be written as:

$$\nabla^2 V=-4\pi\rho,$$

where ρ is the effective density of attracting matter. In modern language, it means that ρ is the difference between the positive and the negative charge density. Poisson also guessed correctly that V is constant over the surface of a conductor. In 1828, Green extended the work of Poisson in electricity and magnetism and gave the name 'potential' to the function V, appearing in Poisson's equation. Poisson also introduced, now what we call, magnetic potential V in terms of the magnetic moment $\vec{m}$ He measured the magnetic moment in terms of the couple required to maintain a magnetic element in equilibrium at a definite angular distance from the magnetic meridian. The scalar potential he discovered is

$$V=\int(\vec{m}\cdot\vec{\nabla})(1/r)\,d^3r$$

This splendid work of Poisson was published in a memoir submitted to the French Academy (1812).

So far, there is no discussion of the passage of charges, i.e. electric current. The motion of charges was discovered quite accidentally by Luigi Galvani in 1780. He observed that the muscles contract when the charges flow through the nerves of a frog. He subsequently discovered that the charges in the thunderstorm as well as the ones produced in the laboratory are transported through the nerves to the muscles. Thus he laid the foundations of what we nowadays call bioelectricity. There was a controversy between him and many others, particularly Volta, regarding the interpretation. We will not go into the controversy but the outcome of this controversy was fantastic, namely, the development of Volta's pile in the year 1800. The pile is simply a battery, a source of electricity. It was made up of copper-wet cloth-zinc-wet cloth-copper types of many layers. Thus modern electrochemistry was born. In the same year, Nicholson and Carlisle made a Volta's pile in England and decomposed water into hydrogen and oxygen. Thus, with Volta, current electricity was born and this accelerated our understanding of electricity.

The relation between electricity and magnetism was a fascinating topic for philosophers as well as for scientists. This was because of some curious effects of lightning brought out through an accident by a tradesman from Wakefield. The tradesman was staying in a hut where he put his knives and forks in a box in a corner. Because of the thunderstorm, a lightning struck. Some knives and forks melted but some got magnetized attracting small nails around. Then, in 1751, Franklin magnetized sewing needles by means of a discharge from a Leyden jar. More experiments followed. It was in 1819, Oersted of Copenhagen showed that a magnetic needle could be deflected when a closed circuit is formed with a voltaic cell. He did not find a quantitative law. It was left to Biot and Savart, and also to Ampére from France to formulate a mathematical connection between the current in the circuit and the magnetic field. Biot and Savart announced their famous law:

$$d\vec{B} = \frac{I}{c}\frac{d\vec{l} \times \vec{r}}{r^3}.$$

In 1820, it was also realized that the field $\vec{B}$ is similar to the magnetic field produced by a natural magnet.

Ampére, within a week's time of the announcement of Oersted's discovery, started experiments to explore the force per unit length between two parallel wires carrying currents and showed that it is

$$F_{ab} = \frac{2I_a I_b}{c^2 R},$$

where I_a and I_b are currents carried by two parallel wires a and b with a perpendicular distance R between them. This experiment is very carefully done even in modern times to define the MKSI unit of current, named after Ampére. He also came to the conclusion that a current-carrying loop acts like a magnetic dipole. He preferred to take a view that the magnetism is due to electrical current and not due to a mysterious magnetic fluid. Certainly this view about magnetism is the current view. He also coined the term electrodynamics. Around 1822 the thermoelectric effect was discovered by Seebeck, establishing a connection between heat and electricity. In 1825, Sir Humphry Davy discovered the concept of resistivity and showed that it is a property of the material. In the year 1826, Ohm discovered his famous law and showed how resistance of a wire can be found if its length, area of cross-section and resistivity are known. Ohm also wrote $\vec{J} = \sigma\vec{E}$, where $\vec{J}$ is current density, $\vec{E}$ is electric field and σ is electrical conductivity.

Many of the mathematical formulae based upon experimental discoveries were given by Neumann and Weber. These were the scientists who believed in 'action at a distance' theory, circuits and so on. 'Fields' were not yet accepted. The last major discoverer in pre Maxwell era was Michael Faraday.[1]

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[1] For more on Faraday, see *Resonance*, Vol.7, No.3, 2002.

In 1812, Humphry Davy received a letter from a young person working as a bookbinder, named Michael Faraday who wanted to escape from the trade he was doing, and join as an assistant to Davy. He had also enclosed a copy of Davy's lectures. Davy interviewed Faraday and gave him a job. By that time, Faraday was 21 and had fabricated Volta's pile and studied chemical decomposition. Faraday was assistant to Davy till 1829 and then became the director of Royal Institution and from 1833, he occupied a special chair of chemistry created for him.

Faraday conceived that, whenever a charge or a magnet is kept, they influence the entire space surrounding it in the form of 'lines of force'. He also found that every line of force from a magnet is closed. Thus Faraday may be called the father of field theory. Because, just like in a field, grass grows everywhere, the magnets or charges influence all points in space.

Discoveries of Oersted and Ampére were known in England and Faraday was influenced by them. His cardinal discovery of the law of induction was made in 1831. He might have wondered that if current produces magnetic induction, can magnetic induction produce current? He set up an experiment with an iron ring with coils of copper wound on it in one-half which connected to a battery and coils of copper wound on the other half with a current measuring device. He thus observed that a current was produced in the circuit whenever there was changing lines of force created in the electromagnet. More the rate of change of magnetic flux cut by the circuit, more current flows. Transformation of this discovery to technology took nearly 50 years when by the end of the nineteenth century, first power stations were installed in London and New York.

Besides his law of induction, he made seminal contributions to electrochemistry. He also discovered how a dielectric behaves in an external electric field. He correctly guessed that, in the process of polarization, equal numbers of positive and negative charges are separated and realized that $\int \vec{\mathcal{J}}\, \rho d^3r = 0$ but $\int \vec{\mathcal{J}}\, \rho d^3r$ is nonzero. He then defined displacement field $\vec{D}$ and also the dielectric constant.

At that time, there was a controversy regarding the propagation of light. It was established that light is a transverse oscillation, which required a medium for its propagation. The existence of such a medium was postulated and was called aether. A large number of physicists and mathematical physicists such as Cauchy, Navier and Lord Kelvin were involved in studying the elastic properties of aether. Faraday published a speculative paper in 1846 titled 'Thoughts of Ray Vibrations'. Faraday speculated that light and heat radiation might be transverse vibrations propagated along lines of force. This certainly is wrong but Faraday abandoned the idea of aether and insisted on the idea of lines of force i.e. fields.

This was essentially the ethos prior to the final discovery of Maxwell, namely detailed formulation of classical electrodynamics and unifying electricity and magnetism, a first step in the grand unification of basic forces of Nature.

Entry of Maxwell

Against the backdrop of several similarities between elasticity, hydrodynamics and electric and magnetic fields becoming clear, and with the understanding of the fields developing, young Maxwell entered the scene. In the year 1856, he sent his first paper on electromagnetism to *Transactions of the Cambridge Philosophical Society*. Here, he compared lines of force of the magnetic and electric fields $\vec{B}$ and $\vec{E}$ to fluid flow. He compared the velocity of a fluid to the lines of force of the electric field in free aether. However, in the presence of many dielectrics of dielectric constant ε, the Faraday's line of force will correspond to $\varepsilon\vec{E}$ With this analogy, he concluded that in free aether with dielectric present,

$$\nabla\cdot(\varepsilon\vec{E})=0.$$

He called $\varepsilon\vec{E}$ as a displacement field vector $\vec{D}$ and concluded that it is circuital (i.e. $\oint \vec{D}.d\vec{l} = 0$) in a charge-free region. A relationship between $\vec{D}$ and $\vec{E}$ is similar to the relationship between $\vec{B}$ and $\vec{H}$ His mathematical formulation heavily used vector ideas of curl and divergence, made popular by Stokes. He rediscovered a vector potential $\vec{A}$ in the same communication, which was known earlier to Neumann, Weber and Kirchhoff. He also realized that the vector potential is not unique but one may have another vector potential gauge equivalent to it. In addition, Maxwell showed in the same memoir that

$$\nabla \times \vec{H} = \frac{4\pi \vec{J}}{c}.$$

In 1858, Helmholtz compared the magnetic field produced by a current with the vortex motion of an incompressible fluid. He showed that the magnetic field corresponds to the fluid velocity and the electric current causing it would then correspond to vortex filament in the fluid. This enabled him to transport many theorems from fluid dynamics to magnetism. In the 1856 memoir, Maxwell hoped, 'By careful study of the laws of elastic solids, and of the motion of viscous fluid, I hope to discover method of forming a mechanical conception of the electrotonic state adapted to general reasoning.'

Comparison of the electrical phenomena with the behaviour of elastic bodies was natural because polarization would be zero when external electric field is zero. This is like a stretched rubber band that takes its original length when a stress on it is removed. The elastic medium was aether. By ascribing different properties to the aether, Riemann, who was a student of Gauss, guessed that changing electrification would cause changes in potential propagating outwards with a speed of light. He guessed there was a need for the modification of the Poisson equation to

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = -4\pi\rho .$$

This was in 1853. But apart from this guess, there was no further advancement by him. Faraday, in 1857 speculated that the velocity of propagation of magnetic disturbance might be the same as that of light in aether, and he questioned as to whether it would get affected if there are 'bodies' in the medium? In other words he asked himself the question as to whether there could be an equation similar to the one guessed by Riemann, for a vector potential with current source? The answer to this question was given by Maxwell in 1861-62 by furnishing a complete theory.

Six years elapsed after Maxwell's first paper. Maxwell was appointed a Professor of natural philosophy in Marischal College, Aberdeen from 1856-60. Afterwards from 1860 to 1865, Maxwell worked in King's College, London.

In 1861 and 1862, Maxwell wrote a series of papers in the *Philosophical Magazine* on electromagnetism to 'devise mechanical conception of the electromagnetic field'.

Faraday had observed that the dielectric polarization increases with increasing external applied field. It is a function of time t if the external applied field is time-dependent. Maxwell might have thought that the electric displacement $\vec{D}$ is proportional to the total free charge. Even though at every instant, total polarization charge is zero, the charges move in a dielectric. If the polarization is a function of time, then $\partial\vec{D}/\partial t$ will be proportional to some kind of a current, which must be added to the conduction current. With suitable units, the effective current is

$$\vec{J}_{\text{eff}} = \vec{J} + \frac{1}{4\pi}\frac{\partial\vec{D}}{\partial t}.$$

Clearly, it is easy to see that $\nabla\cdot\vec{J}_{\text{eff}} = 0$ is an equation of continuity. Maxwell then equated

$$\nabla\times\vec{H} = \frac{4\pi}{c}\vec{J}_{\text{eff}} = \frac{4\pi\vec{J}}{c} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}.$$

This is Maxwell's celebrated modification of Ampére's circuital theorem. Although Maxwell guessed this modification, using an analogy between the behaviour of elastic element and the dielectric, he went a step further. He said that 'there is an electric displacement wherever there is an electric force (i.e. field) whether material bodies are present or not. Thus, he crossed the limit of material bodies of Faraday for electric displacement.

Maxwell then considered the particular case of a dielectric without free charges so that the conduction current is zero. Then,

$$\nabla\cdot\vec{E}=0$$

$$\nabla\cdot\vec{H}=0$$

$$\nabla\times\vec{H}=\frac{\varepsilon}{c}\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}.$$

It follows immediately that

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{bmatrix}\vec{E}\\\vec{H}\end{bmatrix}=0$$

But these are the precise equations, which a light vector would satisfy, propagating with speed c. Moreover, Maxwell showed that the propagation direction of the wave is at right angles to both $\vec{E}$ and $\vec{H}$. He showed that if there is a plane polarized wave moving along z direction, then,

$$cH_y=E_x \text{ and } cH_x=-E_y$$

If there is a dielectric medium of dielectric constant $c/\sqrt{\varepsilon}$, then Maxwell showed that the speed of the electromagnetic (e.m.) wave in the medium will be $c/\sqrt{\varepsilon}$ Maxwell must have been elated. Maxwell then asserted that light is nothing but an electromagnetic wave. The theory also predicted that the refractive index of a medium is nothing but the square root of its dielectric constant. Maxwell developed these views still further, and in 1864, presented a memoir to the Royal Society as 'A Dynamical Theory of the Electromagnetic field'. In this memoir, Maxwell extended his theory to propagation of electromagnetic waves in anisotropic media. Maxwell, then discussed propagation of the e.m. waves in metals, as opposed to the dielectric and arrived at the well-known equation relating to skin depth as

$$\nabla^2 \vec{E} - \frac{\varepsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{4\pi\sigma}{c^2}\frac{\partial \vec{E}}{\partial t} = 0 .$$

This equation is of the same form as the corresponding equation in elastic-solid theory. This equation correctly explains metallic reflection. In 1852, Stokes had analysed the fluorescence phenomenon. He had emphatically stated that the molecules causing the effect had their own natural frequency of vibration. Taking a clue from this idea, Maxwell gave one of the earliest theories of dispersion. Using elastic properties of the medium, he arrived at the dispersion formula as

$$n^2 - 1 = \frac{\text{constant } \nu_0^2}{\nu_0^2 - \nu^2}$$

Where 'n' is refractive index at frequency ν and ν_0 is a natural frequency of the molecules of the medium. This formula is more or less correct in the sense that it does not incorporate damping of oscillations of the molecules. If the system has many natural frequencies, generalization is immediate. In 1897, Rubens verified Maxwell's dispersion formula for KCl and rock salt.

We cannot ignore the contributions of Lorenz (1867) who following Riemann, suggested the modifications for equations of vector and scalar potentials. He introduced the idea of retarded potentials as

$$\phi(\vec{r},t) = \int \frac{\rho(\vec{r}\,',t')}{R} d^3r' \quad \text{and}$$

$$\vec{A}(\vec{r},t) = \frac{1}{c}\int \frac{\vec{J}(\vec{r}\,',t')}{R} d^3r'$$

where $t' = t - (R/c)$ and $R = |\vec{r} - \vec{r}'|$.

He also introduced similar modification of Ampere's circuital theorem. The system, as we now know, gives rise to the equation for vector potential as

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\frac{4\pi \vec{J}}{c},$$

which is exactly similar to the equation for the scalar potential V, correctly guessed by Riemann, but without justification.

Since various colours are nothing but realization of electromagnetic waves in various frequency bands by our eye, Maxwell got interested in the phenomenon of colour perception. He invented the trichromatic process. Using red, green and blue filters he produced the first colour photograph of the Scottish tartar ribbon.

The last major contribution of Maxwell was what is known as 'stress tensor'. Following Faraday, we know that, the lines of force contract (become dense) near the source and repel each other as we go away from the source. Thus there is a lateral pressure on these lines and a tension along the lines. This is similar to the dough used for baking bread.

Maxwell argued that tension along the lines of force would maintain the force acting on a conductor on whose surface the line terminates. The pressure at right angles to the line of force must be such that aether is in equilibrium. Let $\vec{N}$ be the unit normal to the plane in an electromagnetic field across which stresses are to be found. Maxwell showed the stresses as

$$\frac{1}{4\pi}\left\{\left(\vec{D}\cdot\vec{N}\right)\vec{E} + \left(\vec{B}\cdot\vec{N}\right)\vec{H} - \frac{1}{2}\left(\vec{D}\cdot\vec{E} + \vec{B}\cdot\vec{H}\right)\vec{N}\right\}$$

Using the stress tensor arguments, Maxwell predicted that, a beam of light must exert a pressure on a body where it falls. This was confirmed by very careful experiments of Lebedev (1899).

Suggested Reading

[1] Sir. Edmond Whittaker, *A History of the Theories of Aether and Electricity*, Vol. 1 and 2, Thomas Nelson and Sons Ltd. London, 1951.

[2] J C Maxwell, *Treaties on Electricity and Magnetism*, 3rd Edition (1891), Vol. 1 and 2, Dover, New York, 1954.

Maxwell's theory of electromagnetic waves was experimentally verified by David Hughes and independently, and in more detail, by Hertz in 1888. Hertz also conclusively proved that the speed of the wave to be *c*.

Thus, the entire basic superstructure of unification of electricity and magnetism and its relationship with light was established by Maxwell. Hertz, J J Thompson, Lorentz, Poynting, Heaviside, FitzGerald and others extended and used Maxwell's theory to understand various physical phenomena. Even though, the idea of aether had to be abandoned later, the equations stayed intact and Einstein abandoned Newton's laws and preferred Maxwell equations (a difficult choice indeed!) for his relativistic covariance. Thus Maxwell, in a sense, is a first field theorist of repute who united the two basic forces of nature in the last century. We salute him for that.

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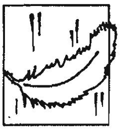

The following letter was addressed to Maxwell by Faraday on receiving a copy of the paper on 'Lines of Force':

Albemarle Street W, 25th March 1857.

MY DEAR SIR

I received your paper, and thank you very much for it. I do not say I venture to thank you for what you have said about 'Lines of Force', because I know you have done it for the interests of philosophical truth; but you must suppose it is work grateful to me, and gives me much encouragement to think on. I was at first almost frightened when I saw such mathematical force made to bear upon the subject, and then wondered to see that the subject stood it so well. I send by this post another paper to you; I wonder what you will say to it. I hope however, that bold as the thoughts may be, you may perhaps find reason to bear with them. I hope this summer to make some experiments on the time of magnetic action, or rather on the time required for the assumption of the electrotonic state, round a wire carrying a current, that may help the subject on. The time must probably be short as the time of light; but the greatness of the result, if affirmative, makes me not despair. Perhaps I had better have said nothing about it, for I am often long in realising my intentions, and a failing memory is against me.

Ever yours most truly,

M. FARADAY